PHYS 320 ANALYTICAL MECHANICS

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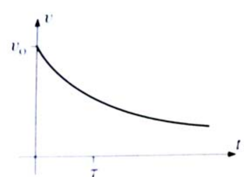
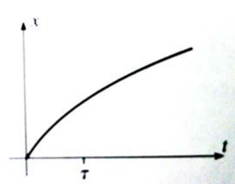
Quadratic Air Resistance: $\vec{f}_{quad} = -c\,v\,\vec{v}$

Allows horizontal solutions

$$v_x(t) = \frac{v_o}{1 + c v_o t / m} = \frac{v_o}{1 + t/\tau}$$

$$x(t) = v_o \tau \ln(1 + t/\tau)$$





Allows vertical solutions

$$v_y(t) = v_{ter} \tanh\left(\frac{gt}{v_{ter}}\right)$$

$$y(t) = \frac{v_{ter}^2}{g} \ln(\cosh\left(\frac{gt}{v_{ter}}\right))$$

$$v_{ter} \equiv \sqrt{\frac{mg}{c}} \qquad \tau \equiv \frac{m}{c v_o}$$

Some mathematics!

$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2} \qquad \cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

Euler's relation: $e^{\pm i\theta} = \cos\theta \pm i\sin\theta$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i} \qquad \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$$

Quadratic Air Resistance: projectiles

$v_{ter} \equiv \sqrt{\frac{mg}{c}}$ $\tau \equiv \frac{m}{cv_o}$

$$\begin{cases} m\dot{v}_x = -c\sqrt{v_x^2 + v_y^2}\ v_x \\ m\dot{v}_y = -mg - c\sqrt{v_x^2 + v_y^2}\ v_y \end{cases}$$

Coupled differential equations!

TRICKY to deal with.
Solve numerically!

Maple with numerical solutions to differential equations

SOLVING DIFFERENTIAL EQUATIONS WITH MAPLE NUMERICALLY:

GWClark

```
> restart; with(plots):g:=9.8;b:=0.1;m:=2;
> eq := m·diff(x(t), t, t) = -g;
> ic := x(0) = 19.6, D(x)(0) = 196;
> soln := dsolve({eq, ic}, x(t), type = numeric);
> odeplot(soln, [t, x(t)], 0..80, labels = [t, x], title = "position vs. time");
> odeplot(soln, [t, diff(x(t), t)], 0..80, labels = [t, v], color = blue, title = "velocity vs. time");
>
```

You have to give Maple some numbers for all the constants!

```
restart : with(plots) :
g := 9.8; m := 1; vo := 20; theta := 45;
```

9.8

1

20

45

Here, c is Taylor's quadratic drag of F[quad]=-cv^2.

```
c := 0.01;
```

0.01

```
ay := diff(y(t), t, t) =-m·g - c·diff(y(t), t)·sqrt(diff(x(t), t)^2 + diff(y(t), t)^2);
```

$$\frac{d^2}{dt^2}y(t) = -9.8 - 0.01\left(\frac{d}{dt}y(t)\right)\sqrt{\left(\frac{d}{dt}x(t)\right)^2 + \left(\frac{d}{dt}y(t)\right)^2}$$

```
ax := diff(x(t), t, t) =-c·diff(x(t), t)·sqrt(diff(x(t), t)^2 + diff(y(t), t)^2)
```

$$\frac{d^2}{dt^2}x(t) = -0.01\left(\frac{d}{dt}x(t)\right)\sqrt{\left(\frac{d}{dt}x(t)\right)^2 + \left(\frac{d}{dt}y(t)\right)^2}$$

```
IC := D(y)(0) = vo·sin(theta·Pi/180), D(x)(0) = vo·cos(theta·Pi/180), y(0) = 0, x(0) = 0;
```

$$D(y)(0) = 10\sqrt{2}, D(x)(0) = 10\sqrt{2}, y(0) = 0, x(0) = 0$$

```
odesys := [ay, ax, IC];
```

$$\left[\frac{d^2}{dt^2}y(t) = -9.8 - 0.01\left(\frac{d}{dt}y(t)\right)\sqrt{\left(\frac{d}{dt}x(t)\right)^2 + \left(\frac{d}{dt}y(t)\right)^2}, \frac{d^2}{dt^2}x(t) = -0.01\left(\frac{d}{dt}x(t)\right)\sqrt{\left(\frac{d}{dt}x(t)\right)^2 + \left(\frac{d}{dt}y(t)\right)^2}, D(y)(0) = 10\sqrt{2}, D(x)(0) = 10\sqrt{2}, y(0) = 0, x(0) = 0\right]$$

```
dsol := dsolve(odesys, numeric);
```

proc(x_rkf45) ... **end proc**

```
P1 := odeplot(dsol, [x(t), y(t)], 0..2.7, color = blue);
```